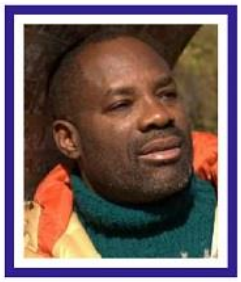


Philip Emeagwali
info@emeagwali.com 202-203-8724

My Quest for an Internet

In the sixty-ninth installment of our weekly series at emeagwali.com, we focus on Emeagwali's 1989 discovery of 72 mathematical and algorithmic terms that corrected a 150-year-old error in eighteen critical equations used to extract oil. Those terms made the news when he solved them on two-to-power sixteen, or 65,536, computers that were connected by sixteen times two-to-power sixteen, or 1,048,576, communication wires as a superinternet (a word that Emeagwali coined) in a metaphorical sixteen dimensional universe.

Making Strides in a 16 Dimensional Universe

Profile of Philip Emeagwali in a leading oil and gas monthly published in Norway. Up to date videos are posted at emeagwali.com.

Booking Request

Page 22 UPSTREAM PROFILE 27 January 1997

The awards keep flowing for computer wizard Dr Philip Emeagwali. The unorthodox innovator has pushed back the boundaries of oilfield science but life in America is tainted by 'pervasive racism'

BARRY MORGAN
from Accra

NIGERIAN-born Dr Philip Emeagwali has an enviable reputation in the oil industry for his achievements in computing, gathering a hatful of awards from around the world.

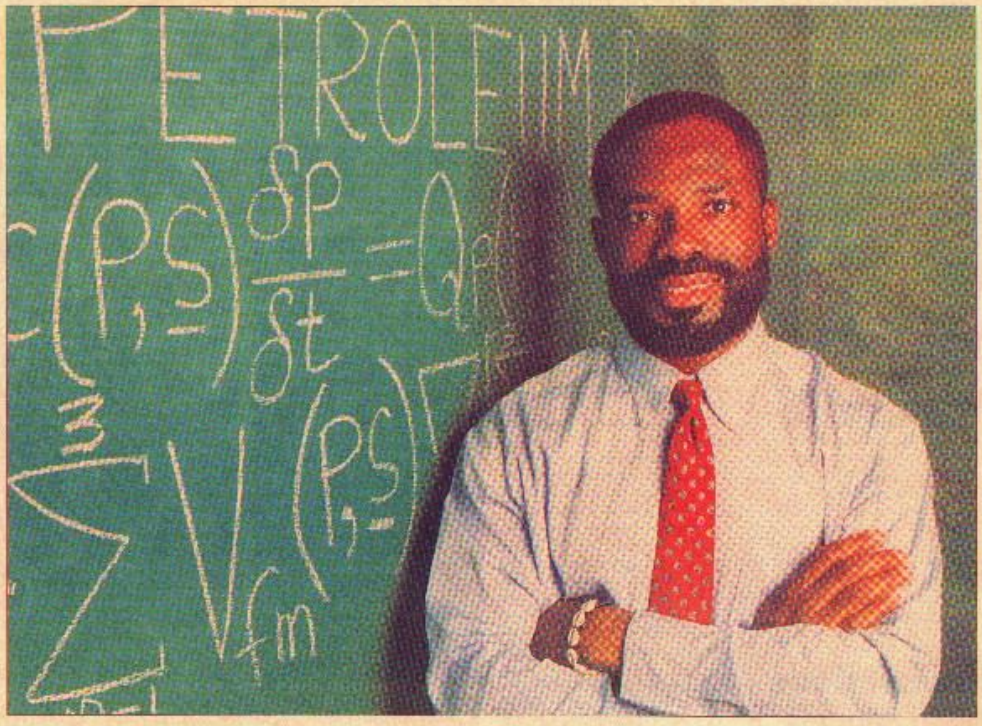
He first shot to fame in 1989 when he won the coveted Gordon Bell Prize for his work with massively parallel computers. He programmed a Cray Connection Machine to compute a world record 3.1 billion calculations per second using 65,536 processors to simulate oil reservoirs.

He has been busy recently on work that could lead to a better understanding of reservoir mechanics and enhance field recovery, and the laurels keep coming.

His home country awarded the Baltimore-based graduate of Michigan University the prestigious 1996 Nigeria Prize, Africa's top scholarly prize, while America's National Society of Black Engineers named him Pioneer of the Year.

At 41, Emeagwali has the aura of a renaissance man and is highly conscious of the example he sets to the younger generation of African American scholars.

"I work very long hours. However, I am not an obsessed scien-



Natural talent: Emeagwali says research and development at the big oil companies is conventional and rational "while my approach is multi-disciplinary, unorthodox, intuitive and nature-inspired"

Nature's own numbers man

Inspiration from hard history

THE DOCTOR and his wife Dale, a fellow academic,

is multi-disciplinary, unorthodox, intuitive and nature-inspired

Nature's own numbers man

Michigan University the prestigious 1996 Nigeria Prize, Africa's top scholarly prize, while America's National Society of Black Engineers named him Pioneer of the Year.

At 41, Emeagwali has the aura of a renaissance man and is highly conscious of the example he sets to the younger generation of African American scholars.

"I work very long hours. However, I am not an obsessed scientist. As long as I am making discoveries I will continue conducting research," he says. "Otherwise I will change to a field that will use some of the transferable skills that I have acquired."

Emeagwali says the oil industry has been progressive in adapting new technologies, noting that one in 10 supercomputers is bought for use in oilfield simulation. "But at the big companies, research and development is conventional, rational and mechanic," he laments, "while my research approach is multi-disciplinary, unorthodox, intuitive and nature-inspired."

He believes oil companies should look beyond their current reservoir simulators. "I will use my nature-inspired study of the evolution of plants over the past 400 million years and the optimised branching structure of trees to design new algorithms and computers that can be used to discover and recover more oil," he says.

Seeing similarities between flow patterns in reservoirs and in the oceans and atmosphere, Emeagwali claims that coming from a low-tech African environment gives him a greater appreciation of the inspiration that can be drawn from nature.

"My new discoveries are so wacky that I might be called crazy. For example, I have found that algorithms, software and computers can be enantiomeric — that is, they have left-handed and right-handed versions like shoes. I would like to apply these ideas to practical problems in the petroleum industry."

He has just formulated a mosaic-like modelling theory for parallel computing along with a new concept for better utilising the fast calculations needed to find and recover more oil.

Emeagwali is proud of his success "as an unorthodox researcher", tying together hard-won expertise in science, mathematics, engineering and computing.

He says of his Cray breakthrough: "Stubborn determination was needed to continue pursuing a line of enquiry which other scientists considered crazy or foolish. No one would collaborate with me or sponsor my work but I refused to give up."

His dogged perseverance paid off. The good doctor will not put a figure on it but was amused by rumours at the time that he was worth $200 million and was the "Bill Gates of Africa".

But Emeagwali would rather be seen pursuing the Nobel Prize than a fat bank account. He sees himself as an innovator. "The new problem-solving approach of designing computer networks by observing and emulating patterns in nature is one that I pioneered. Mine is a logistic and inspirational approach," he says.

He attributes his success to his Igbo background in south-east Nigeria and a "can do" spirit. These are traits he wants to pass on to his young son "who is going to encounter racism in the US which will deny him the opportunity to contribute as much as he can to society.

"I want my son to be inspired by the fact that I was a high school drop-out and ex-refugee who overcame racism and made scientific contributions that benefited mankind."

Making strides in a parallel universe

OIL COMPANIES are now pouring money into massively parallel computers — proof, says Emeagwali, that the technology has broken through initial industry scepticism.

His prize-winning work in 1989 enabled him to develop the first "pseudo-time" approach to reservoir modelling. It proved to the US establishment that "the use of only Dirichlet-type equations in the vicinity of petroleum production wells located near the boundary is better suited to avoid the coning problems caused by the high velocity of converging flows in the vicinity of wells".

The work goes to the heart of a debate that raged in the 1930s over equations governing porous media flow modelling, and which this doughty scientist now sees good reason to revive.

Reservoir models using Darcy's 19th century equations can no longer be held valid near pumping wells with a turbulent flow regime, he claims. He also discovered the analogy between Darcy's equations and geostrophic equations used in weather forecasting, something his industry counterparts still find hard to accept.

Uprooting old models will, he says, be costly and difficult. "Reservoir models have taken hundreds of man years and tens of millions of dollars to construct and it will be impractical to discard them. But they can be modified to incorporate the additional physics used by my new equations." He does not feel his ideas have been under-utilised by the industry and points out that his original goal was simply to demonstrate that massively parallel computers can be used to discover and recover additional oil and gas.

The next challenge, he says, will require a mix of technologies with faster computers that can perform one trillion calculations per second — a feat, Emeagwali predicts, that will be achieved in less than two years.

Inspiration from hard history

THE DOCTOR and his wife Dale, a fellow academic, lead an annual scientific workshop for inner-city youths and he runs a website (http://acm.org/~philip) used as a teaching resource in US classrooms.

The mayor of a Minnesota town appointed him to a Shelter Board, which helps the homeless, and he serves on the board of the African Health Organisation.

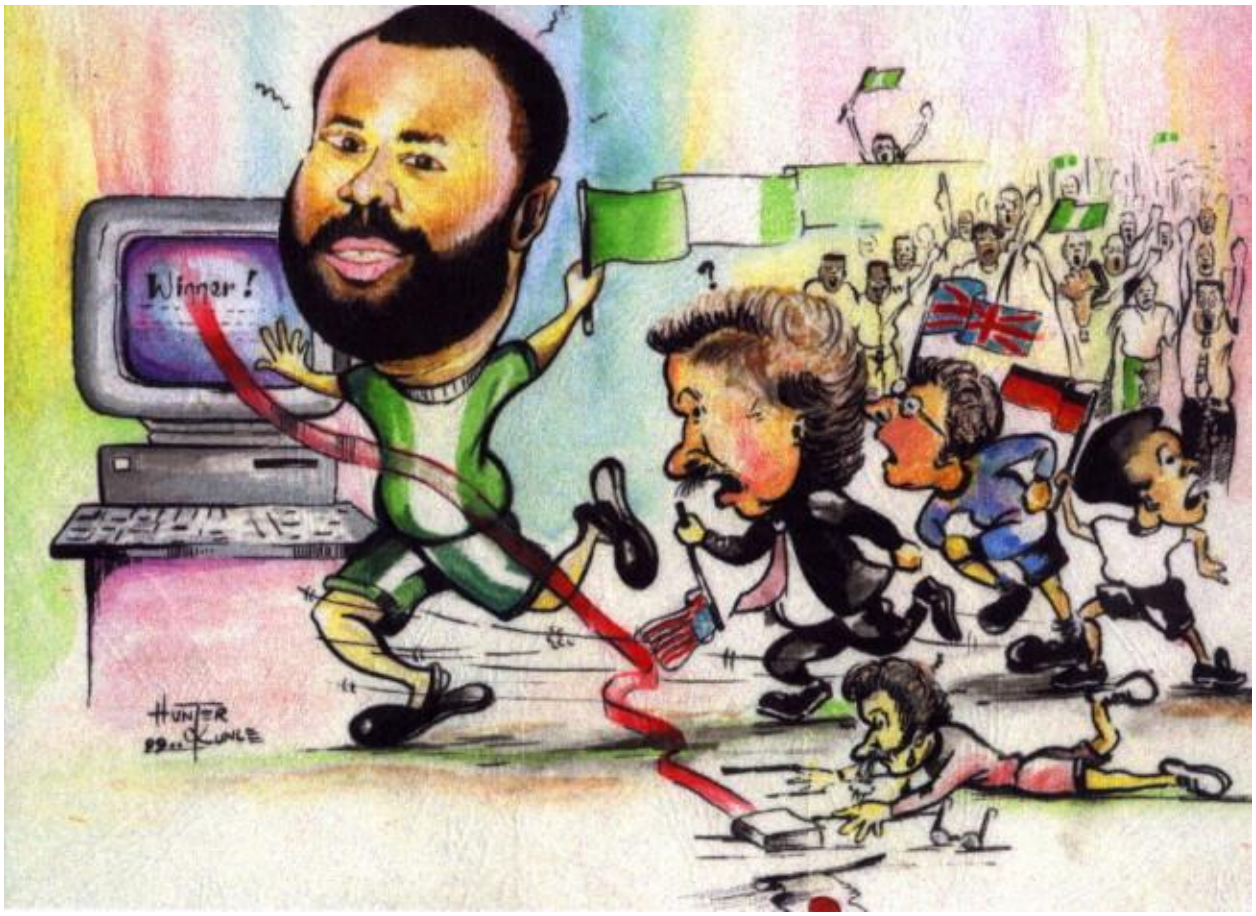
Emeagwali was inspired by the 18th century letters of black mathematician Benjamin Bannekar to Thomas Jefferson.

He says: "Bannekar's amazing mathematical calculations so impressed Jefferson that he was forced to review his opinion of the intellectual capacity of Africans and urged they be freed from slavery."

Emeagwali himself appears to be making a career of overturning conventional wisdom. The scientist is currently researching a book on how the Greeks learned mathematics from Africans.

But despite his successes and evident satisfaction with his professional life, he has strong reservations about "pervasive and deeply ingrained racism" in America.

He says: "The US is one of the worst countries to live in for an ambitious black professional."


Winner!

$$(1): \quad \frac{\partial C_i}{\partial t} + \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = q, \quad i = o, w, g$$

The fluxes:

$$(2): \quad (\vec{J})_o = \frac{\rho_{osc}}{B_o} \vec{V}_o$$

$$(\vec{J})_w = \frac{\rho_{wsc}}{B_w} \vec{V}_w$$

$$(\vec{J})_g = \frac{\rho_{gsc}}{B_g} \vec{V}_g + \frac{R_{so} \rho_{gsc}}{B_o} \vec{V}_o + \frac{R_{sw} \rho_{gsc}}{B_w} \vec{V}_w$$

o, w, and g = oil, water, and gas, resp.
sc = standard conditions, approx. 14.7 psia & 60°F
ρ = densities
B = formation volume factor
R_{so}, R_{sw} = gas solubities
$\vec{V}$ = Darcian velocities

$$(3_x): \quad V_{xo} = -K_x \lambda_o \frac{\partial}{\partial x}\left[P_o - \frac{\rho_o g z}{144 g_c}\right]$$

$$V_{xw} = -K_x \lambda_w \frac{\partial}{\partial x}\left[P_w - \frac{\rho_w g z}{144 g_c}\right]$$

$$V_{xg} = -K_x \lambda_g \frac{\partial}{\partial x}\left[P_g - \frac{\rho_g g z}{144 g_c}\right]$$

g_c = constant, $\approx$ 32 ft/sec^2
g = gravitational acceleration

Similarly, for the y and z directions

$$V_{yo} = -K_y \lambda_o \frac{\partial}{\partial y}\left[P_o - \frac{\rho_o g z}{144 g_c}\right]$$

$$(3_y): \quad V_{yw} = -K_y \lambda_w \frac{\partial}{\partial y}\left[P_w - \frac{\rho_w g z}{144 g_c}\right]$$

$$V_{yg} = -K_y \lambda_g \frac{\partial}{\partial y}\left[P_g - \frac{\rho_g g z}{144 g_c}\right]$$

$$(3_z): \quad V_{zo} = -K_z \lambda_o \frac{\partial}{\partial z}\left[P_o - \frac{\rho_o g z}{144 g_c}\right]$$

$$V_{zw} = -K_z \lambda_w \frac{\partial}{\partial z}\left[P_w - \frac{\rho_w g z}{144 g_c}\right]$$

$$V_{zg} = -K_z \lambda_g \frac{\partial}{\partial z}\left[P_g - \frac{\rho_g g z}{144 g_c}\right]$$

where

λ_i = phase mobility = K_{ri}/μ_i

where

K_{ri} = relative permeability

μ_i = viscosity

$i = o, w, \& g,$ [redacted]

For the oil, water and gas phases, the Darcian velocities are

$$\vec{V}_i = -\underset{\approx}{K} \frac{K_{ri}}{\mu_i} \nabla \Phi_i$$

where

$\underset{\approx}{K}$: permeability tensor, diagonal along principal axes

μ_i : viscosities of oil, water, gas

K_{ri} : relative permeabilities of oil, water, gas

The phase potential $\Phi = \Phi(z)$, z = depth

$$\Phi_o = P_o - \frac{\rho_o z}{144}$$

$$\Phi_w = P_o - P_{cow} - \frac{\rho_w z}{144}$$

$$\Phi_g = P_o + P_{cgo} - \frac{\rho_g z}{144}$$

The compressibilities for the rock and oil, water and gas are:

$$C_o = -\left[\frac{1}{B_o}\frac{\partial B_o}{\partial P_o} - \frac{B_g}{B_o}\frac{\partial R_{so}}{\partial P_o}\right]$$

$$C_w = -\left[\frac{1}{B_w}\frac{\partial B_w}{\partial P_o} - \frac{B_g}{B_w}\frac{\partial R_{sw}}{\partial P_o}\right]$$

$$C_g = -\frac{1}{B_g}\frac{\partial B_g}{\partial P_o}$$

$$C_r = \frac{1}{\Phi}\frac{\partial \Phi}{\partial P_o}$$

(from the input PVT data)

The densities of the oil, water and gas phases are dependent on the solubilities of the gases and on the formation volume factors. Accordingly,

$$\rho_o = \frac{1}{B_o}\left[\rho_{osc} + R_{so}\rho_{gsc}\right]$$

$$\rho_w = \frac{1}{B_w}\left[\rho_{wsc} + R_{sw}\rho_{gsc}\right]$$

$$\rho_g = \frac{\rho_{gsc}}{B_g}$$

The CONCENTRATIONS are dependent on porosity (ϕ) and saturation (S):

$$C_o = \phi\,\rho_{osc}\,S_o/B_o$$

(4): $$C_w = \phi\,\rho_{wsc}\,S_w/B_w$$

$$C_g = \phi\,\rho_{gsc}\left[\frac{S_g}{B_g} + R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w}\right]$$

By definition:

$$S_o + S_w + S_g = 1$$

Eqns. 1, 2, 3, 4, yields

$$\frac{\partial}{\partial t}\left(\phi \rho_{osc} \frac{S_o}{B_o}\right) + q_o + \left[\frac{\partial}{\partial x}\left(\frac{\rho_{osc}}{B_o} V_{xo}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{osc}}{B_o} V_{yo}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{osc}}{B_o} V_{zo}\right)\right] = 0$$

$$\frac{\partial}{\partial t}\left(\phi \rho_{wsc} \frac{S_w}{B_o}\right) + q_w + \left[\frac{\partial}{\partial x}\left(\frac{\rho_{wsc}}{B_w} V_{xw}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{wsc}}{B_w} V_{yw}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{wsc}}{B_w} V_{zw}\right)\right] = 0$$

$$\frac{\partial}{\partial t}\left[\phi \rho_{gsc}\left(\frac{S_g}{B_g} + \frac{R_{so} S_o}{B_o} + \frac{R_{sw} S_w}{B_w}\right)\right] + q_g + \frac{\partial}{\partial x}\left(\frac{\rho_{gsc}}{B_g} V_{xg} + \frac{R_{so}\rho_{gsc}}{B_o} V_{xo} + \frac{R_{sw}\rho_{gsc}}{B_w} V_{xw}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{gsc}}{B_g} V_{yg} + \frac{R_{so}\rho_{gsc}}{B_o} V_{yo} + \frac{R_{sw}\rho_{gsc}}{B_w} V_{yz}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{gsc}}{B_g} V_{zg} + \frac{R_{so}\rho_{gsc}}{B_o} V_{zo} + \frac{R_{sw}\rho_{gsc}}{B_w} V_{zw}\right) = 0$$

Assume that both the temperature and pressure are constant within the oilfield.

Then,

$$\rho_{osc}\frac{\partial}{\partial t}\left(\phi\frac{S_o}{B_o}\right)+q_o+\frac{\partial}{\partial x}\left(\frac{V_{xo}}{B_o}\right)+\frac{\partial}{\partial y}\left(\frac{V_{yo}}{B_o}\right)+\frac{\partial}{\partial z}\left(\frac{V_{zo}}{B_o}\right)=0$$

$$\rho_{wsc}\frac{\partial}{\partial t}\left(\phi\frac{S_w}{B_w}\right)+q_w+\frac{\partial}{\partial x}\left(\frac{V_{xw}}{B_w}\right)+\frac{\partial}{\partial y}\left(\frac{V_{yz}}{B_w}\right)+\frac{\partial}{\partial z}\left(\frac{V_{zw}}{B_w}\right)=0$$

$$\rho_{gsc}\frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g}+R_{so}\frac{S_o}{B_o}+R_{sw}\frac{S_w}{B_w}\right)\right]+q_g+\frac{\partial}{\partial x}\left(\frac{V_{xg}}{B_g}+\frac{R_{so}}{B_o}V_{xo}+\frac{R_{sw}}{B_w}V_{xw}\right)+\frac{\partial}{\partial y}\left(\frac{V_{yg}}{B_g}+\frac{R_{so}}{B_o}V_{yo}+\frac{R_{sw}}{B_w}V_{yw}\right)+\frac{\partial}{\partial z}\left(\frac{V_{zg}}{B_g}+\frac{R_{so}}{B_o}V_{zo}+\frac{R_{sw}}{B_w}V_{zw}\right)=0$$

Define

$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x} V_x + \frac{\partial}{\partial y} V_y + \frac{\partial}{\partial z} V_z$$

Then

$$\frac{\partial}{\partial t}\left(\phi \frac{S_o}{B_o}\right) + \nabla \cdot \frac{\vec{V}_o}{B_o} + \frac{q_o}{\rho_{osc}} = 0$$

$$\frac{\partial}{\partial t}\left(\phi \frac{S_w}{B_w}\right) + \nabla \cdot \frac{\vec{V}_w}{B_w} + \frac{q_w}{\rho_{wsc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g} + R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w}\right)\right]$$

$$+ \nabla \cdot \left(\frac{\vec{V}_g}{B_g} + \frac{R_{so}}{B_o}\vec{V}_o + \frac{R_{sw}}{B_w}\vec{V}_w\right) + \frac{q_g}{\rho_{gsc}} = 0$$

We derived NINE Darcian velocities for oil, water & gas, along x, y, z directions;
THREE phase mobility equations for oil, water, & gas;
THREE phase densities for oil, water and gas;
ONE saturation constraint;
THREE conservation of mass equations for oil, water and gas.

The oil wells are represented by the source/sink term

$$Q_K$$

The Darcian velocities for the oil, water, and gas phases are

$$V_i = -\underline{K}\,\frac{K_{ri}}{\mu_i}\cdot\left(\nabla P_i - \gamma_i \nabla z\right)$$

Over time, the rates at which the oil, water and gas accumulates are:

$$\frac{\partial}{\partial t}\left[\phi \sum_{i=1}^{n_p} \rho_i x_{Ki} S_i\right]$$

Over space, the rates at which the oil, water and gas are convected are:

$$\nabla\cdot\left[\sum_{i=1}^{n_p} \rho_i x_{Ki} V_i\right]$$

When gas injection is miscible, the rates of dispersion for the oil, water and gas are

$$\nabla\cdot\left[\sum_{i=1}^{n_p} \phi S_i \underset{\approx}{D}_{Ki}\, \rho_i \cdot \nabla x_{Ki}\right]$$

where

$\underset{\approx}{K}$ & $\underset{\approx}{D}_{Ki}$: permeability, dispersion tensors, resp.

ϕ : porosity

K_{ri} : relative permeability for oil, water, gas

n_p : number of phases

n_c : number of components

V_i : Darcian velocities

S_i, P_i : saturations, pressures for oil, water, gas

ρ_i, μ_i : densities, viscosities for oil, water, gas

γ_i : pressure gradients for oil, water, gas,

x_{Ki} : Mole fraction of component K in oil, water, gas

IMPES Approximation
IMplicit Pressure
Explicit Saturation

In vector notation, the conservation of mass equations for oil, water and gas are:

$$\frac{\partial}{\partial t}\left[\phi \frac{S_o}{B_o}\right] + \nabla \cdot \frac{\vec{V}_o}{B_o} + \frac{q_o}{\rho_{osc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi \frac{S_w}{B_w}\right] + \nabla \cdot \frac{\vec{V}_w}{B_w} + \frac{q_w}{\rho_{wsc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{R_{so}}{B_o} S_o + \frac{R_{sw}}{B_w} S_w + \frac{1}{B_g} S_g\right)\right]$$

$$+ \nabla \cdot \left[\frac{R_{so}}{B_o} \vec{V}_o + \frac{R_{sw}}{B_w} \vec{V}_w + \frac{1}{B_g} \vec{V}_g\right]$$

$$+ \frac{q_g}{\rho_{gsc}} = 0$$

where

ϕ : porosity

V : Darcian velocities for oil, water, gas

R_{sw} : solubility of gas in water

R_{so} : solubility of gas in oil

ρ_{sc} : densities of oil, water, gas at standard conditions

B : formation volume factors for oil, water, gas

q : mass flow rates per unit of reservoir volume for oil, water, gas.

S : saturation for oil, water, gas

SATURATION CONSTRAINT

$$S_o + S_w + S_g = 1$$

CAPILLARY PRESSURE

$$P_{cow}(S_w) = P_o - P_w$$

$$P_{cgo}(S_g) = P_g - P_o$$

where

P : pressures for oil, water, gas

P_{cow} : oil-water capillary pressure

P_{cgo} : gas-oil capillary pressure

DARCIAN VELOCITIES

$$\vec{V}_i = -K \frac{K_{ri}}{\mu_i} \nabla \phi_i$$

for oil, water, gas.

Generally, the permeability tensor, K, is diagonalized along its principal axes.

$\vec{V}$: Darcian velocities for oil, water, gas

μ : viscosity for oil, water, gas

K_r : relative permeability for oil, water, gas

ϕ : the phase potentials for oil, water, gas

PHASE POTENTIALS

The phase potential varies ~~with~~ along z-direction (i.e. depth) and is defined as:

$$\phi_o = P_o - \frac{\rho_o z}{144}$$

$$\phi_w = P_o - P_{cow} - \frac{\rho_w z}{144}$$

$$\phi_g = P_o + P_{cgo} - \frac{\rho_g z}{144}$$

PHASE DENSITIES

We use the PVT data (input) to compute the three phase densities as:

$$\rho_o = \frac{1}{B_o}\left[R_{so}\rho_{gsc} + \rho_{osc}\right]$$

$$\rho_w = \frac{1}{B_w}\left[R_{sw}\rho_{gsc} + \rho_{wsc}\right]$$

$$\rho_g = \frac{1}{B_g}\left[\rho_{gsc}\right]$$

COMPRESSIBILITIES

$$C_o = \frac{1}{B_o}\left[B_g\frac{\partial R_{so}}{\partial P_o} - \frac{\partial B_o}{\partial P_o}\right]$$

$$C_w = \frac{1}{B_w}\left[B_g\frac{\partial R_{sw}}{\partial P_o} - \frac{\partial B_w}{\partial P_o}\right]$$

$$C_g = -\frac{1}{B_g}\frac{\partial B_g}{\partial P_o}$$

$$C_r = \frac{1}{\phi}\frac{\partial \phi}{\partial P_o}$$

The preceding set of equations are solved by discretizing the pressure implicitly and the saturation explicitly. as defined in:

Sawyer, W. K. and Mercer, J. C., "Applied Simulation Techniques for Energy Recovery," Department of Energy Report METC/RI-78/9, Morgantown, WV, 1978.

DARCIAN VELOCITIES

$$\vec{V}_o = -\overleftrightarrow{K} \cdot \lambda_o \nabla \Phi_o$$

$$= -\lambda_o \left[\hat{i} K_x \frac{\partial \Phi_o}{\partial x} + \hat{j} K_y \frac{\partial \Phi_o}{\partial y} + \hat{k} K_z \frac{\partial \Phi_o}{\partial z} \right]$$

$$\vec{V}_w = -\overleftrightarrow{K} \cdot \lambda_w \nabla \Phi_w$$

$$= -\lambda_w \left[\hat{i} K_x \frac{\partial \Phi_w}{\partial x} + \hat{j} K_y \frac{\partial \Phi_w}{\partial y} + \hat{k} K_z \frac{\partial \Phi_w}{\partial z} \right]$$

$$\vec{V}_g = -\overleftrightarrow{K} \cdot \lambda_g \nabla \Phi_g$$

$$= -\lambda_g \left[\hat{i} K_x \frac{\partial \Phi_g}{\partial x} + \hat{j} K_y \frac{\partial \Phi_g}{\partial y} + \hat{k} K_z \frac{\partial \Phi_g}{\partial z} \right]$$

where

$\overleftrightarrow{K}$: permeability tensor of rank two, written in dyadic notation. We assumed that the reference coordinate axes are aligned to those of the permeability tensor.

$\hat{i}, \hat{j}, \hat{k}$: unit vectors

POTENTIAL

The potential for the oil, gas, and water phases are

$$\phi_i = P_i - \frac{\rho_i z}{144}, \text{ where } g = g_c$$

CONSERVATION OF MASS

$$\frac{\partial}{\partial t}\left[\phi \frac{S_o}{B_o}\right] + \nabla \cdot \frac{\vec{V}_o}{B_o} + \frac{q_o}{\rho_{osc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi \frac{S_w}{B_w}\right] + \nabla \cdot \frac{\vec{V}_w}{B_w} + \frac{q_w}{\rho_{wsc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi\left(R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w} + \frac{S_g}{B_g}\right)\right]$$

$$+ \nabla \cdot \left(\frac{R_{so}}{B_o}\vec{V}_o + \frac{R_{sw}}{B_w}\vec{V}_w + \frac{\vec{V}_g}{B_g}\right)$$

$$+ \frac{q_g}{\rho_{gsc}} = 0$$

where

$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x} V_x + \frac{\partial}{\partial y} V_y + \frac{\partial}{\partial z} V_z$$

PARABOLIC EQUATIONS

The preceding Darcian velocities and conservation of mass equations are equivalent to

$$\frac{\partial}{\partial t}\left[\frac{\phi S_o}{B_o}\right] = \nabla \cdot \frac{\overleftrightarrow{K}\lambda_o}{B_o} \cdot \nabla \Phi_o - \frac{q_o}{\rho_{osc}}$$

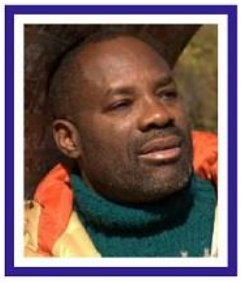
$$\frac{\partial}{\partial t}\left[\frac{\phi S_w}{B_w}\right] = \nabla \cdot \frac{\overleftrightarrow{K}\lambda_w}{B_w} \cdot \nabla \Phi_w - \frac{q_w}{\rho_{wsc}}$$

$$\frac{\partial}{\partial t}\left[\phi\left(R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w} + \frac{S_g}{B_g}\right)\right]$$

$$= \nabla \cdot \overleftrightarrow{K} \cdot \left[\frac{R_{so}\lambda_o}{B_o}\nabla\Phi_o + \frac{R_{sw}\lambda_w}{B_w}\nabla\Phi_w + \frac{\lambda_g}{B_g}\nabla\Phi_g\right] - \frac{q_g}{\rho_{gsc}}$$

HYPERBOLIC EQUATIONS

The nine equations derived by Emeagwali are used, instead of Darcy's formula. These nine equations are solved simultaneously with the conservation of mass and several other equations.



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