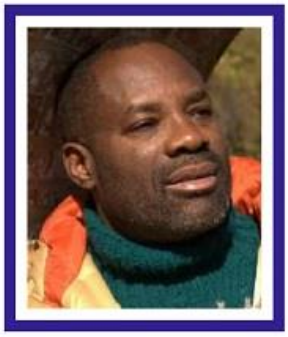

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Quest for Internet

In the fourth installment of our weekly series at emeagwali.com, we focus on Emeagwali's 1989 discovery of 72 mathematical and algorithmic terms that corrected a 150-year-old error in eighteen critical equations used to extract oil. Those terms made the news when he solved them on 65,536 computers that were connected by 1,048,576 communication wires as a superinternet (a word that Emeagwali coined.)

They Call Me "Calculus"

Transcribed from a lecture delivered by Philip Emeagwali. The unedited video is posted at emeagwali.com.

Booking Request

It was June 1970. The place was Venn Road, Onitsha, Nigeria.

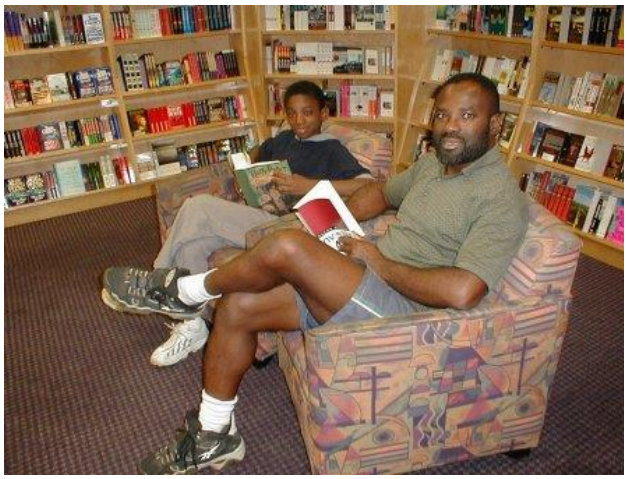
The elderly woman said:

"I don't know who you're talking about."

"Don't you know the young boy that walks around with a big book in his hand?" her companion asked.

"Ooooh Hoooo!!! Now I know who you're talking about."

I was "the young boy." Then 15 years old and at Christ the King College, Onitsha. I had retired from the Biafran army five months earlier at Ndoni, including a brief trip to the Oguta War Front in July 1969.



My wife (Dale) takes our son to the public library every Saturday. Here I (reading "*Invisible Man*") and him (Ijeoma reading "*Harry Potter*") in a bookstore. She calls the public library "shamefully underutilized." (Baltimore, Maryland. October 21, 2000)

Everybody in this photo called me "Calculus." (Emeagwali: In dark shirt and 7th from right of last row. Saint Charles House, Christ the King College, Onitsha, Nigeria. 1971)

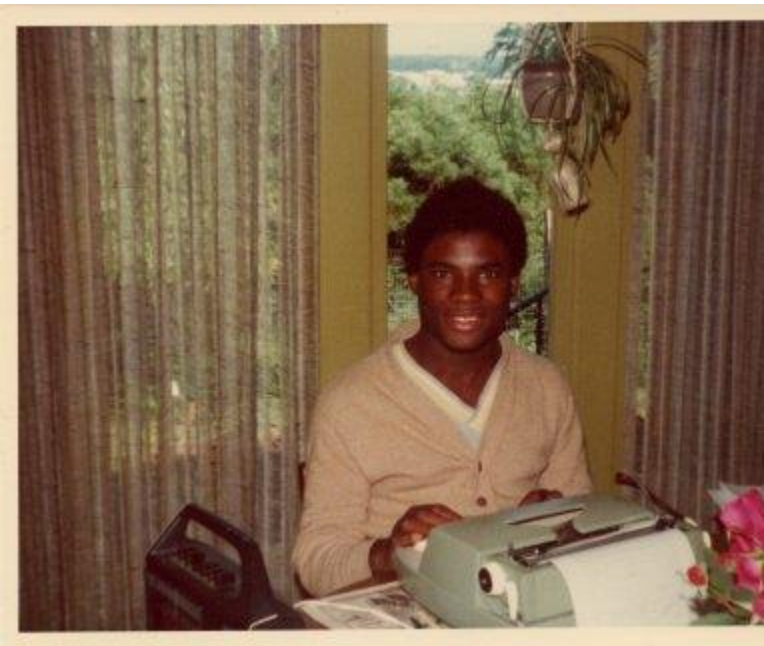
My peers called me "Calculus." And the book that I carried around and that gave me the nickname "Calculus" was a 568-page blue hard bound called "*An Introduction to the Infinitesimal Calculus, With Applications to Mechanics and Physics*" by George William Caunt.

In 1970, all 15-year-olds in Onitsha had dropped out of school for three years to live in a refugee camp. So our mathematical maturity was actually that of a 12-year-old. And the equations in that book was quite heart-stopping. Certainly, it was anything but introductory for classmates that were still in chapter three of high school algebra by C.V. Durel at the academically rigorous all boys Catholic boarding school known as C.K.C.

A close-up of me (behind, dark shirt) from the above photo.

At first, I was called "Infinitesimal Calculus." But the word infinitesimal was a tongue twister amongst the illiterate Onitsha traders and they shortened it to Calculus.

I was also known in faraway corners of Nigeria, by word of mouth and by media such as the *Daily Times* (the nation's only truly national newspaper) that positively mentioned my name in its Sunday Science Column. I was then seventeen and hoping to get a scholarship and a visa to the United States.



My research began in 1974 in the Pacific Northwest region of the United States.(Philip Emeagwali, 15 NW Edgewood Drive, Corvallis, Oregon. May 1975)

Fast forward a decade, I was attending daily mathematics seminars in nearby College

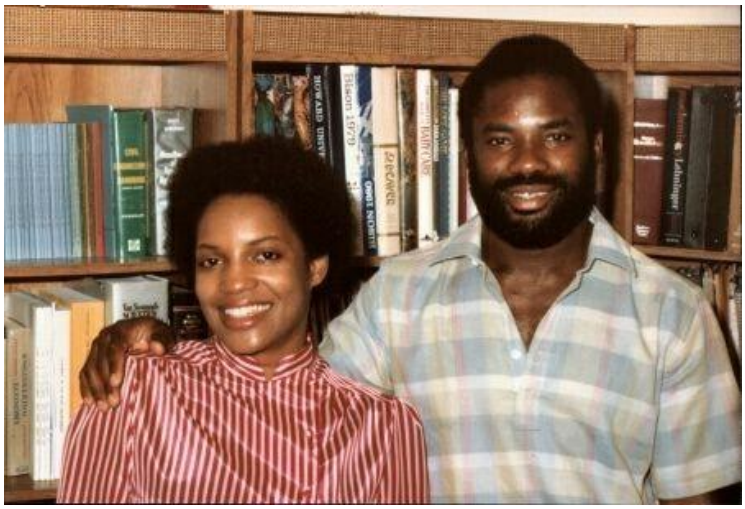
My 36 terms were embodied in the nine partial differential equations I invented. These were announced as a headline story in the May 1990 issue of *SIAM News*, the flagship publication of mathematicians.

Park, Maryland, a small town in the outskirts of Washington that is a mecca amongst mathematicians from far away countries.

After ten years, I was at a level that I don't carry maths books around. Instead, I carried blank yellow pads because the equations that I needed were not in any textbooks. I had to create or invent them.

For the record, my first two airplane rides were spurred by two mathematics scholarships. And the next two airplane rides after that were to deliver lectures to mathematicians on the new calculus that I invented that pushed the frontier of partial differential equations (advanced calculus) by a distance of 72 partial derivative terms.

For more than 130 years, no mathematician had noticed the error in the petroleum industry's central equation, Darcy's formula. Although countless mathematicians had reviewed the formula since its 1856 inception, it wasn't until 1989 that anyone caught and reported the error—36 missing mathematical terms called partial derivatives.



A 1983 photo at home in Silver Spring, Maryland, with my wife Dale and my books on Darcy's formula. I discovered and corrected a critical error in this formula, the most important in geology,

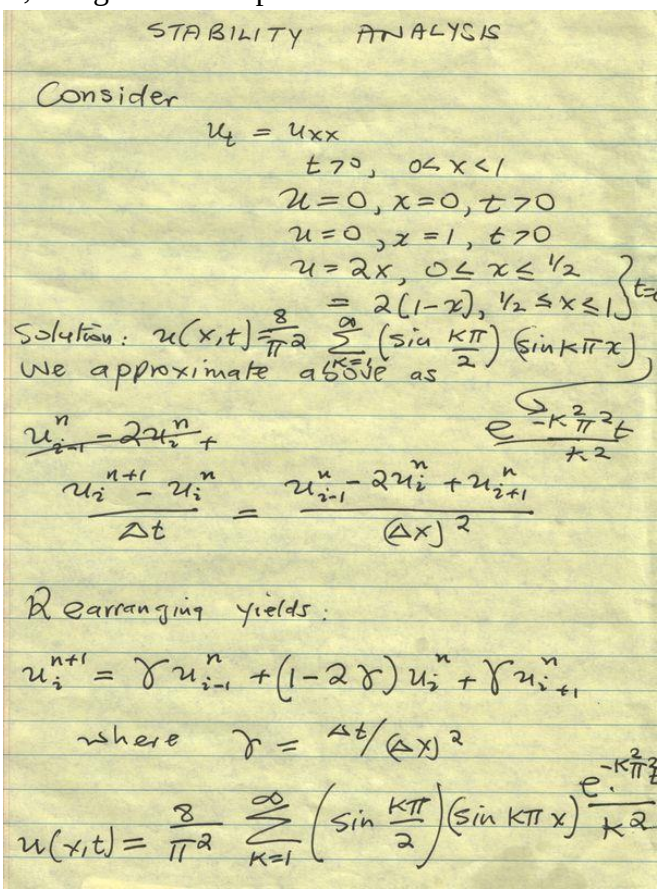
This error went undetected because it was small in a physical sense. However, it is not small in a mathematical, algorithmic, or computational sense. Most importantly, in an industry that earns trillions of dollars in oil revenue a year, even a tiny error can cost a billion dollars.

While previous reviews of the equation were mathematically thorough, none appeared to have re-examined the physics used to derive the equations. When I re-examined them, I discovered that an error occurred in the manner that the physics were used to formulate the mathematics. I corrected it at its source, the Second Law of Motion. Because the Second Law was being violated, I discovered that the equations,

A peek into my mathematical notes, circa 1983.

algorithms, and codes derived from it were physics defying.

In 1989, I announced my discovery of 36 partial derivatives for oil recovery that corrected these mathematical errors. Each of my 36 terms represented the components of temporal and convective inertial forces, for oil, water, and gas within a petroleum



STABILITY ANALYSIS

Consider

$$u_t = u_{xx}$$

$$t > 0, \quad 0 < x < 1$$

$$u = 0, \; x = 0, \; t > 0$$

$$u = 0, \; x = 1, \; t > 0$$

$$u = 2x, \; 0 \le x \le 1/2$$

$$= 2(1-x), \; 1/2 \le x \le 1$$

Solution: $u(x,t) = \frac{8}{\pi^2} \sum_{k=1}^{\infty} \left(\sin \frac{k\pi}{2}\right) (\sin k\pi x) \frac{e^{-k^2\pi^2 t}}{k^2}$

We approximate above as

$$\cancel{u_{i-1}^n - 2u_i^n +}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{u_{i-1}^n - 2u_i^n + u_{i+1}^n}{(\Delta x)^2}$$

Rearranging yields:

$$u_i^{n+1} = \gamma u_{i-1}^n + (1-2\gamma) u_i^n + \gamma u_{i+1}^n$$

where $\gamma = \Delta t / (\Delta x)^2$

$$u(x,t) = \frac{8}{\pi^2} \sum_{k=1}^{\infty} \left(\sin \frac{k\pi}{2}\right) (\sin k\pi x) \frac{e^{-k^2\pi^2 t}}{k^2}$$

reservoir. I discovered them because no mathematician had previously applied the 36 terms to a reservoir simulation. I also invented 36 algorithms to solve the algorithmic and simulation errors that occur when and where the 36 derivatives are missing.

I explained this exciting application to mathematicians, both in writing and in person. At seminars, I would scribble the actual equations used by Exxon to simulate the flow of oil, water, and gas inside its reservoirs. I would then point out that the textbooks, Exxon and all oil companies only sum three forces: pressure, viscous, and gravity.

Finally, I would re-derive the nine correct partial differential equations from the Second Law of Motion, ensuring that each contained four partial derivative terms that represented a fourth force called 'inertia.' The laws of physics are to mathematical physics what the constitution is to the law. It was obvious that my equations were more accurate than what was in the textbooks, simulators, and supercomputers because mine accounted for all four forces that exist inside every reservoir.

My 36 terms were embodied in the nine partial differential equations I invented. These were announced as a headline story in the May 1990 issue of *SIAM News*. This is the flagship publication of the

Society for Industrial and Applied Mathematics (SIAM), the premier society for applied mathematicians.

Thanks to that publicity, I was invited to present my equations at the International Congress on Industrial and Applied Mathematics (ICIAM). This conference is held every four years and is the largest gathering of mathematicians in the world. I accepted their invitation because it was the best forum in which to explain to the field's foremost experts how my equations pushed the frontier of calculus by a distance of 36 partial derivative inertial terms, which are discoveries for fluids flowing below the surface of the earth. I presented my nine equations and nine algorithms that embodied my 72 terms on July 8, 1991, in Washington, D.C.

I explained that I re-introduced the "temporal inertial force" and the "convective inertial force" into nine partial differential equations that define the three-dimensional motion of oil, water, and gas within a petroleum reservoir simulator. Incorporating inertia meant that I had to re-derive the nine partial differential equations that are coupled as a system. I also had to invent nine algorithms that I used to formulate 24 million algebraic equations—a world record in 1989—that I solved on two-to-power sixteen or 65,536 computers that were connected by sixteen times two-to-power sixteen or 1,048,576 bi-directional communication wires as a superinternet.

My equations are akin to the iconic Navier-Stokes equations. The main difference is that mine only apply to porous media or fluids that flow underneath the earth, while the Navier-Stokes equations are for fluids above it. That means that my equations are for simulating oilfields while the primitive equations (akin to Navier-Stokes) are used to simulate the weather. I also announced that my equations are hyperbolic, while the incorrect ones were parabolic.

I

refer to my equations in the plural because they are a system of nine equations that are coupled with other well-known equations. The nine algorithms I invented to solve these nine equations were also coupled. Altogether, I filled six blackboards with 18 complex equations and algorithms. The simplest among them look like the strange hieroglyphics of numbers and Greek symbols from my early 1980s research notes on Navier-Stokes equations (shown at the bottom). Despite its complexity, mathematicians understood the depth of meaning hidden within my symbols,

The laws of physics are to mathematical physics what the constitution is to the law. It was obvious that my equations were more accurate than what was in the textbooks.

namely, it's a calculus restatement of the Second Law of Motion.

The equations I invented are mathematically equivalent to the iconic Navier-Stokes equations. That should not come as a surprise because both are based on different but physically equivalent problems. They are both describing the same reality but for two domains: fluids flowing below and above the earth's surface.

Despite its complexity and stature as one of the seven Millennium Problems of mathematics, the Navier-Stokes equation is simply a sequence of symbols split into left and right sides joined by an equal sign—a more esoteric way of saying the F=ma in high school physics textbooks.

Put differently, the mathematical seed of both the Millennium Problem that is based on the Navier-Stokes equations and the Grand Challenge Problem of accurately simulating an oil reservoir is: the sum of the forces is equal to mass times acceleration.. I reduced F=ma to a recurrence formula, in which each of my 24 million unknown terms, evenly distributed amongst two-to-power 16 (or 65,536) computers that were connected as a 16-dimensional superinternet, is a function of a set of 24 million known terms.

Additionally, my finite difference algorithms for solving my partial differential equations (PDE) are far more complex than the PDE. I had to invent those algorithms because a computer cannot solve a partial differential equation in its raw "infinite" form.

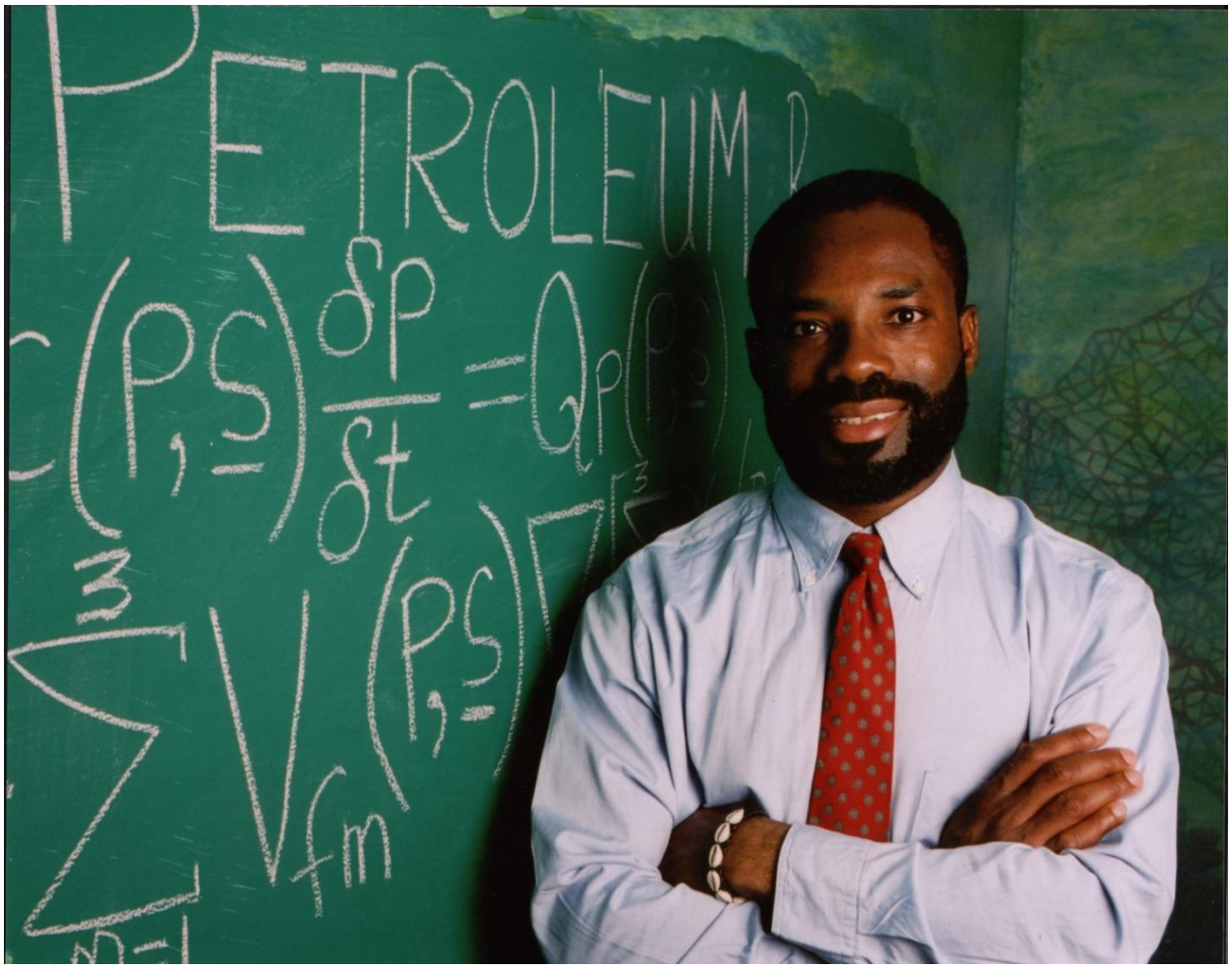
I announced at the International Congress that I had discovered that in a petroleum reservoir simulation, the left side (which corresponds to the temporal and convective inertial forces) was missing. Therefore, the equal sign was *de facto* removed. I also discovered the equivalent of the left side to increase the accuracy with which geologists could simulate reservoirs and, thus, increase the amount of oil recovered.

You have to be a renaissance person and a polymath to solve unsolved problems in mathematics. For this Grand Challenge Problem, you have to be a quadruple threat that is equally at home in physics, mathematics, computation, and communication. The deeper I understood the internet, the closer I moved to the supercomputer that is connected as an internet. And the more I programmed 65,536 sub-computers that defined my supercomputer, the closer I moved to

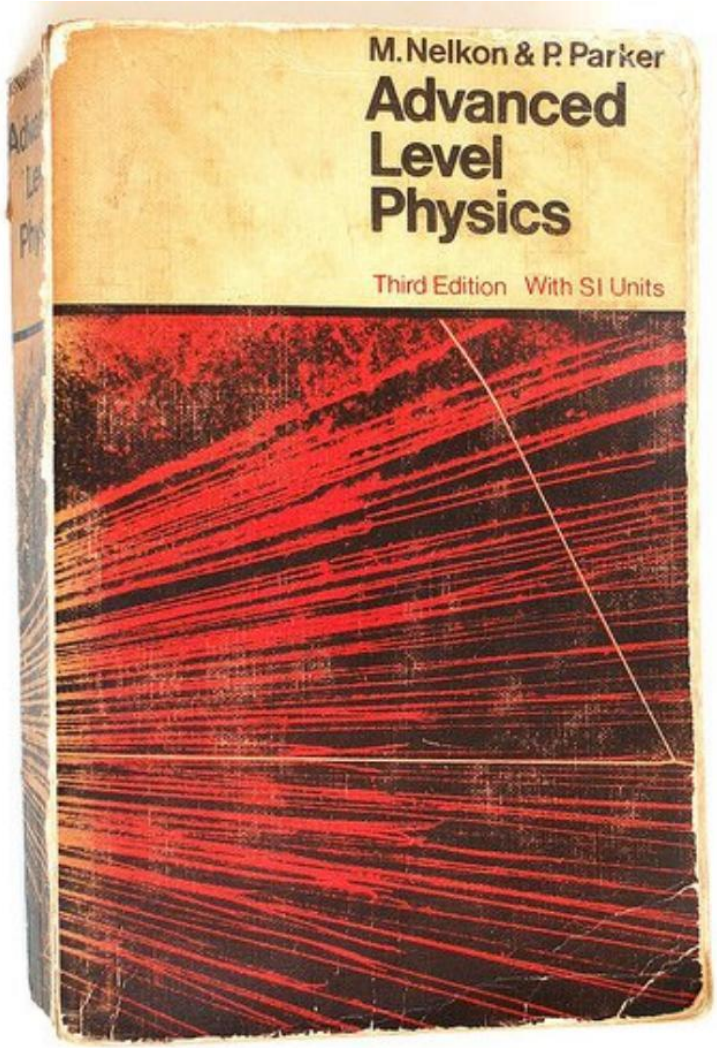
mathematics. And finally to physics where I discovered that to stand on only three forces where four exist is like standing on quicksand, In other words, if four forces exist, four forces are the only firm ground to stand on. Thus solving some unsolved problems in mathematics requires additional discoveries in physics, computation, and communication. For this reason, a ground breaking discovery is the product, not the sum, of lesser discoveries. The highest accomplishment in science is to discover something bigger than oneself.

My photo at the time I invented my equations. I was then (1977-86) interacting with mathematicians in College Park, Maryland, a renown mecca amongst the Who's Who in Mathematics. (Baltimore, Maryland. December 1982)

Despite its complexity, mathematicians understood the depth of meaning hidden within my symbols, namely, it's a calculus restatement of the Second Law of Motion.



I write on the board the actual equations used by Exxon (now Exxon Mobil) to simulate the flow of oil, water, and gas inside its reservoirs. I discovered that their equations did not reflect reality and corrected their error. [emeagwali.com]



I learned the Second Law of Motion from this book, as a self taught student.

The first step is to use an advanced form of calculus called partial differential equations to reduce the statement that "matter cannot be created or destroyed" into a set of mathematical equations with several unknowns. Depending on the complexity of the formulations, we may get anywhere from four to two dozen unknowns. If I have four unknowns, then I must have four equations. Otherwise, it will be impossible for me (or my computer) to solve my equations. Similarly, two dozen unknowns requires two dozen equations.

The law of conservation of momentum gave me nine partial differential equations - one for each of the three primary spatial directions of our universe and one for the oil, water and gas. The law of conservation of matter yields another partial differential equation. The law of conservation of energy yields another equation. I also obtain additional equations by incorporating some well known and established principles of matter and thermodynamics.

Again, my scribblings shown in the images below might look like a strange hieroglyphics of numbers and symbols. What I am doing is using the language of mathematics to derive new and powerful results and make predictions that are consequences of the axioms and postulates of

mathematics which also satisfy the underlying universal laws of science.

One of these laws, is called Second Law of Motion. I reformulated it as nine coupled partial differential equations. The Law of Conservation of Momentum states that the total amount of motion within an oilfield remains the same, irrespective of the number of pumping and injection wells.

The Second Law of Motion, without the fourth of four forces, was integrated with the Laws of Conservation of oil and gas and their equations of states to obtain the following:

$$\nabla \cdot \left[\lambda_o K \left(\nabla P_o - \rho_o g \right) \right] = \frac{\partial}{\partial t} \left(\frac{\Phi S_o}{B_o} \right) + q_o, \quad (1)$$

oil phase pressure; oil density; volumetric production rate per unit oil reservoir volume ~~that is negative~~

$$\nabla \cdot \left[\lambda_w K \left(\nabla P_w - \rho_w g \right) \right] = \frac{\partial}{\partial t} \left(\frac{\Phi S_w}{B_w} \right) + q_w, \quad (2)$$

permeability which is a function of $\underline{x}$; gravitational acceleration vector; The constant rock porosity; water phase saturation; negative when injecting water into reservoir.

$$t > 0, \quad \underline{x} = (x, y, z) \in \Omega,$$

Two P.D.E.s (above); Four Unknowns S_o, S_w, P_o, P_w; Use Two functional relationships below to solve PDE's saturation and pressure distributions.

$$S_o + S_w = 1, \quad (3)$$

$$P_{cow} = P_o - P_w = f(S_w), \quad (4)$$

The functional relationship between water saturation and the capillary pressure.

Use "Simultaneous Solution Method" to solve ALL four equations, boundary and initial conditions

I.Cs. $P_L(x, y, z, 0) = P_L^o(x, y, z)$, $L = o, w$, $\underline{x} \in \Omega$

~~B.Cs.~~ ~~∂P_L~~ Use (3) and (4) to remove saturations in (1) and (2). which then contains ONLY oil pressure (P_o) and water pressure (P_w)

2-11H24

$$(1): \quad \frac{\partial C_i}{\partial t} + \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = q, \quad i = o, w, g$$

The fluxes:

$$(2): \quad (\vec{J})_o = \frac{\rho_{osc}}{B_o} \vec{V}_o$$

$$(\vec{J})_w = \frac{\rho_{wsc}}{B_w} \vec{V}_w$$

$$(\vec{J})_g = \frac{\rho_{gsc}}{B_g} \vec{V}_g + \frac{R_{so}\rho_{gsc}}{B_o} \vec{V}_o + \frac{R_{sw}\rho_{gsc}}{B_w} \vec{V}_w$$

o, w, and g = oil, water, and gas, resp.
sc = standard conditions, approx. 14.7 psia & 60°F
ρ = densities
B = formation volume factor
R_{so}, R_{sw} = gas solubities
$\vec{V}$ = Darcian velocities

$$(3_x): \quad V_{xo} = -K_x \lambda_o \frac{\partial}{\partial x}\left[P_o - \frac{\rho_o g z}{144 g_c}\right]$$

$$V_{xw} = -K_x \lambda_w \frac{\partial}{\partial x}\left[P_w - \frac{\rho_w g z}{144 g_c}\right]$$

$$V_{xg} = -K_x \lambda_g \frac{\partial}{\partial x}\left[P_g - \frac{\rho_g g z}{144 g_c}\right]$$

g_c = constant, $\approx$ 32 ft/sec^2
g = gravitational acceleration

Similarly, for the y and z directions

$$V_{yo} = -K_y \lambda_o \frac{\partial}{\partial y}\left[P_o - \frac{\rho_o g z}{144 g_c}\right]$$

$$(3_y): \quad V_{yw} = -K_y \lambda_w \frac{\partial}{\partial y}\left[P_w - \frac{\rho_w g z}{144 g_c}\right]$$

$$V_{yg} = -K_y \lambda_g \frac{\partial}{\partial y}\left[P_g - \frac{\rho_g g z}{144 g_c}\right]$$

$$(3_z): \quad V_{zo} = -K_z \lambda_o \frac{\partial}{\partial z}\left[P_o - \frac{\rho_o g z}{144 g_c}\right]$$

$$V_{zw} = -K_z \lambda_w \frac{\partial}{\partial z}\left[P_w - \frac{\rho_w g z}{144 g_c}\right]$$

$$V_{zg} = -K_z \lambda_g \frac{\partial}{\partial z}\left[P_g - \frac{\rho_g g z}{144 g_c}\right]$$

where

λ_i = phase mobility = K_{ri}/μ_i

where

K_{ri} = relative permeability

μ_i = viscosity

$i = o, w, \& g,$ [redacted]

For the oil, water and gas phases, the Darcian velocities are

$$\vec{V}_i = -\underset{\approx}{K} \frac{K_{ri}}{\mu_i} \nabla \Phi_i$$

where

$\underset{\approx}{K}$: permeability tensor, diagonal along principal axes

μ_i : viscosities of oil, water, gas

K_{ri} : relative permeabilities of oil, water, gas

The phase potential $\Phi = \Phi(z)$, z = depth

$$\Phi_o = P_o - \frac{\rho_o z}{144}$$

$$\Phi_w = P_o - P_{cow} - \frac{\rho_w z}{144}$$

$$\Phi_g = P_o + P_{cgo} - \frac{\rho_g z}{144}$$

The compressibilities for the rock and oil, water and gas are:

$$C_o = -\left[\frac{1}{B_o}\frac{\partial B_o}{\partial P_o} - \frac{B_g}{B_o}\frac{\partial R_{so}}{\partial P_o}\right]$$

$$C_w = -\left[\frac{1}{B_w}\frac{\partial B_w}{\partial P_o} - \frac{B_g}{B_w}\frac{\partial R_{sw}}{\partial P_o}\right]$$

$$C_g = -\frac{1}{B_g}\frac{\partial B_g}{\partial P_o}$$

$$C_r = \frac{1}{\Phi}\frac{\partial \Phi}{\partial P_o}$$

(from the input PVT data)

The densities of the oil, water and gas phases are dependent on the solubilities of the gases and on the formation volume factors. Accordingly,

$$\rho_o = \frac{1}{B_o}\left[\rho_{osc} + R_{so}\rho_{gsc}\right]$$

$$\rho_w = \frac{1}{B_w}\left[\rho_{wsc} + R_{sw}\rho_{gsc}\right]$$

$$\rho_g = \frac{\rho_{gsc}}{B_g}$$

The CONCENTRATIONS are dependent on porosity (ϕ) and saturation (S):

$$C_o = \phi\,\rho_{osc}\,S_o/B_o$$

(4): $$C_w = \phi\,\rho_{wsc}\,S_w/B_w$$

$$C_g = \phi\,\rho_{gsc}\left[\frac{S_g}{B_g} + R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w}\right]$$

By definition:

$$S_o + S_w + S_g = 1$$

Eqns. 1, 2, 3, 4, yields

$$\frac{\partial}{\partial t}\left(\phi \rho_{osc} \frac{S_o}{B_o}\right) + q_o + \left[\frac{\partial}{\partial x}\left(\frac{\rho_{osc}}{B_o} V_{xo}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{osc}}{B_o} V_{yo}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{osc}}{B_o} V_{zo}\right)\right] = 0$$

$$\frac{\partial}{\partial t}\left(\phi \rho_{wsc} \frac{S_w}{B_o}\right) + q_w + \left[\frac{\partial}{\partial x}\left(\frac{\rho_{wsc}}{B_w} V_{xw}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{wsc}}{B_w} V_{yw}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{wsc}}{B_w} V_{zw}\right)\right] = 0$$

$$\frac{\partial}{\partial t}\left[\phi \rho_{gsc}\left(\frac{S_g}{B_g} + \frac{R_{so} S_o}{B_o} + \frac{R_{sw} S_w}{B_w}\right)\right] + q_g + \frac{\partial}{\partial x}\left(\frac{\rho_{gsc}}{B_g} V_{xg} + \frac{R_{so}\rho_{gsc}}{B_o} V_{xo} + \frac{R_{sw}\rho_{gsc}}{B_w} V_{xw}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{gsc}}{B_g} V_{yg} + \frac{R_{so}\rho_{gsc}}{B_o} V_{yo} + \frac{R_{sw}\rho_{gsc}}{B_w} V_{yz}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{gsc}}{B_g} V_{zg} + \frac{R_{so}\rho_{gsc}}{B_o} V_{zo} + \frac{R_{sw}\rho_{gsc}}{B_w} V_{zw}\right) = 0$$

Assume that both the temperature and pressure are constant within the oilfield.

Then,

$$\rho_{osc} \frac{\partial}{\partial t}\left(\phi \frac{S_o}{B_o}\right) + q_o + \frac{\partial}{\partial x}\left(\frac{V_{xo}}{B_o}\right) + \frac{\partial}{\partial y}\left(\frac{V_{yo}}{B_o}\right) + \frac{\partial}{\partial z}\left(\frac{V_{zo}}{B_o}\right) = 0$$

$$\rho_{wsc} \frac{\partial}{\partial t}\left(\phi \frac{S_w}{B_w}\right) + q_w + \frac{\partial}{\partial x}\left(\frac{V_{xw}}{B_w}\right) + \frac{\partial}{\partial y}\left(\frac{V_{yz}}{B_w}\right) + \frac{\partial}{\partial z}\left(\frac{V_{zw}}{B_w}\right) = 0$$

$$\rho_{gsc} \frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g} + R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w}\right)\right] + q_g + \frac{\partial}{\partial x}\left(\frac{V_{xg}}{B_g} + \frac{R_{so}}{B_o}V_{xo} + \frac{R_{sw}}{B_w}V_{xw}\right) + \frac{\partial}{\partial y}\left(\frac{V_{yg}}{B_g} + \frac{R_{so}}{B_o}V_{yo} + \frac{R_{sw}}{B_w}V_{yw}\right) + \frac{\partial}{\partial z}\left(\frac{V_{zg}}{B_g} + \frac{R_{so}}{B_o}V_{zo} + \frac{R_{sw}}{B_w}V_{zw}\right) = 0$$

Define

$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x} v_x + \frac{\partial}{\partial y} v_y + \frac{\partial}{\partial z} v_z$$

Then

$$\frac{\partial}{\partial t}\left(\phi \frac{S_o}{B_o}\right) + \nabla \cdot \frac{\vec{V}_o}{B_o} + \frac{q_o}{\rho_{osc}} = 0$$

$$\frac{\partial}{\partial t}\left(\phi \frac{S_w}{B_w}\right) + \nabla \cdot \frac{\vec{V}_w}{B_w} + \frac{q_w}{\rho_{wsc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g} + R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w}\right)\right]$$

$$+ \nabla \cdot \left(\frac{\vec{V}_g}{B_g} + \frac{R_{so}}{B_o}\vec{V}_o + \frac{R_{sw}}{B_w}\vec{V}_w\right) + \frac{q_g}{\rho_{gsc}} = 0$$

We derived NINE Darcian velocities for oil, water & gas, along x, y, z directions;
THREE phase mobility equations for oil, water, & gas;
THREE phase densities for oil, water and gas;
ONE saturation constraint;
THREE conservation of mass equations for oil, water and gas.

The oil wells are represented by the source/sink term

$$Q_K$$

The Darcian velocities for the oil, water, and gas phases are

$$V_i = -\underline{K}\,\frac{K_{ri}}{\mu_i}\cdot\left(\nabla P_i - \gamma_i \nabla z\right)$$

Over time, the rates at which the oil, water and gas accumulates are:

$$\frac{\partial}{\partial t}\left[\phi \sum_{i=1}^{n_p} \rho_i x_{Ki} S_i\right]$$

Over space, the rates at which the oil, water and gas are convected are:

$$\nabla\cdot\left[\sum_{i=1}^{n_p} \rho_i x_{Ki} V_i\right]$$

When gas injection is miscible, the rates of dispersion for the oil, water and gas are

$$\nabla\cdot\left[\sum_{i=1}^{n_p} \phi S_i \underset{\approx}{D}_{Ki}\, \rho_i \cdot \nabla x_{Ki}\right]$$

where

$\underset{\approx}{K}$ & $\underset{\approx}{D}_{Ki}$: permeability, dispersion tensors, resp.

ϕ : porosity

K_{ri} : relative permeability for oil, water, gas

n_p : number of phases

n_c : number of components

V_i : Darcian velocities

S_i, P_i : saturations, pressures for oil, water, gas

ρ_i, μ_i : densities, viscosities for oil, water, gas

γ_i : pressure gradients for oil, water, gas,

x_{Ki} : Mole fraction of component K in oil, water, gas

IMPES Approximation
IMplicit Pressure
Explicit Saturation

In vector notation, the conservation of mass equations for oil, water and gas are:

$$\frac{\partial}{\partial t}\left[\phi \frac{S_o}{B_o}\right] + \nabla \cdot \frac{\vec{V}_o}{B_o} + \frac{q_o}{\rho_{osc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi \frac{S_w}{B_w}\right] + \nabla \cdot \frac{\vec{V}_w}{B_w} + \frac{q_w}{\rho_{wsc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{R_{so}}{B_o} S_o + \frac{R_{sw}}{B_w} S_w + \frac{1}{B_g} S_g\right)\right]$$

$$+ \nabla \cdot \left[\frac{R_{so}}{B_o} \vec{V}_o + \frac{R_{sw}}{B_w} \vec{V}_w + \frac{1}{B_g} \vec{V}_g\right]$$

$$+ \frac{q_g}{\rho_{gsc}} = 0$$

where

ϕ : porosity

V : Darcian velocities for oil, water, gas

R_{sw} : solubility of gas in water

R_{so} : solubility of gas in oil

ρ_{sc} : densities of oil, water, gas at standard conditions

B : formation volume factors for oil, water, gas

q : mass flow rates per unit of reservoir volume for oil, water, gas.

S : saturation for oil, water, gas

SATURATION CONSTRAINT

$$S_o + S_w + S_g = 1$$

CAPILLARY PRESSURE

$$P_{cow}(S_w) = P_o - P_w$$

$$P_{cgo}(S_g) = P_g - P_o$$

where

P : pressures for oil, water, gas

P_{cow} : oil-water capillary pressure

P_{cgo} : gas-oil capillary pressure

DARCIAN VELOCITIES

$$\vec{V}_i = -K \frac{K_{ri}}{\mu_i} \nabla \phi_i$$

for oil, water, gas.

Generally, the permeability tensor, K, is diagonalized along its principal axes.

$\vec{V}$: Darcian velocities for oil, water, gas

μ : viscosity for oil, water, gas

K_r : relative permeability for oil, water, gas

ϕ : the phase potentials for oil, water, gas

PHASE POTENTIALS

The phase potential varies ~~with~~ along z-direction (i.e. depth) and is defined as:

$$\phi_o = P_o - \frac{\rho_o z}{144}$$

$$\phi_w = P_o - P_{cow} - \frac{\rho_w z}{144}$$

$$\phi_g = P_o + P_{cgo} - \frac{\rho_g z}{144}$$

PHASE DENSITIES

We use the PVT data (input) to compute the three phase densities as:

$$\rho_o = \frac{1}{B_o}\left[R_{so}\rho_{gsc} + \rho_{osc}\right]$$

$$\rho_w = \frac{1}{B_w}\left[R_{sw}\rho_{gsc} + \rho_{wsc}\right]$$

$$\rho_g = \frac{1}{B_g}\left[\rho_{gsc}\right]$$

COMPRESSIBILITIES

$$C_o = \frac{1}{B_o}\left[B_g\frac{\partial R_{so}}{\partial P_o} - \frac{\partial B_o}{\partial P_o}\right]$$

$$C_w = \frac{1}{B_w}\left[B_g\frac{\partial R_{sw}}{\partial P_o} - \frac{\partial B_w}{\partial P_o}\right]$$

$$C_g = -\frac{1}{B_g}\frac{\partial B_g}{\partial P_o}$$

$$C_r = \frac{1}{\phi}\frac{\partial \phi}{\partial P_o}$$

The preceding set of equations are solved by discretizing the pressure implicitly and the saturation explicitly. as defined in:

Sawyer, W. K. and Mercer, J. C., "Applied Simulation Techniques for Energy Recovery," Department of Energy Report METC/RI-78/9, Morgantown, WV, 1978.

DARCIAN VELOCITIES

$$\vec{V}_o = -\overleftrightarrow{K} \cdot \lambda_o \nabla \Phi_o$$

$$= -\lambda_o \left[\hat{i} K_x \frac{\partial \Phi_o}{\partial x} + \hat{j} K_y \frac{\partial \Phi_o}{\partial y} + \hat{k} K_z \frac{\partial \Phi_o}{\partial z} \right]$$

$$\vec{V}_w = -\overleftrightarrow{K} \cdot \lambda_w \nabla \Phi_w$$

$$= -\lambda_w \left[\hat{i} K_x \frac{\partial \Phi_w}{\partial x} + \hat{j} K_y \frac{\partial \Phi_w}{\partial y} + \hat{k} K_z \frac{\partial \Phi_w}{\partial z} \right]$$

$$\vec{V}_g = -\overleftrightarrow{K} \cdot \lambda_g \nabla \Phi_g$$

$$= -\lambda_g \left[\hat{i} K_x \frac{\partial \Phi_g}{\partial x} + \hat{j} K_y \frac{\partial \Phi_g}{\partial y} + \hat{k} K_z \frac{\partial \Phi_g}{\partial z} \right]$$

where

$\overleftrightarrow{K}$: permeability tensor of rank two, written in dyadic notation. We assumed that the reference coordinate axes are aligned to those of the permeability tensor.

$\hat{i}, \hat{j}, \hat{k}$: unit vectors

POTENTIAL

The potential for the oil, gas, and water phases are

$$\phi_i = P_i - \frac{\rho_i z}{144}, \text{ where } g = g_c$$

CONSERVATION OF MASS

$$\frac{\partial}{\partial t}\left[\phi \frac{S_o}{B_o}\right] + \nabla \cdot \frac{\vec{V}_o}{B_o} + \frac{q_o}{\rho_{osc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi \frac{S_w}{B_w}\right] + \nabla \cdot \frac{\vec{V}_w}{B_w} + \frac{q_w}{\rho_{wsc}} = 0$$

$$\frac{\partial}{\partial t}\left[\phi\left(R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w} + \frac{S_g}{B_g}\right)\right]$$

$$+ \nabla \cdot \left(\frac{R_{so}}{B_o}\vec{V}_o + \frac{R_{sw}}{B_w}\vec{V}_w + \frac{\vec{V}_g}{B_g}\right)$$

$$+ \frac{q_g}{\rho_{gsc}} = 0$$

where

$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x} V_x + \frac{\partial}{\partial y} V_y + \frac{\partial}{\partial z} V_z$$

PARABOLIC EQUATIONS

The preceding Darcian velocities and conservation of mass equations are equivalent to

$$\frac{\partial}{\partial t}\left[\frac{\phi S_o}{B_o}\right] = \nabla \cdot \frac{\overleftrightarrow{K}\lambda_o}{B_o} \cdot \nabla \Phi_o - \frac{q_o}{\rho_{osc}}$$

$$\frac{\partial}{\partial t}\left[\frac{\phi S_w}{B_w}\right] = \nabla \cdot \frac{\overleftrightarrow{K}\lambda_w}{B_w} \cdot \nabla \Phi_w - \frac{q_w}{\rho_{wsc}}$$

$$\frac{\partial}{\partial t}\left[\phi\left(R_{so}\frac{S_o}{B_o} + R_{sw}\frac{S_w}{B_w} + \frac{S_g}{B_g}\right)\right]$$

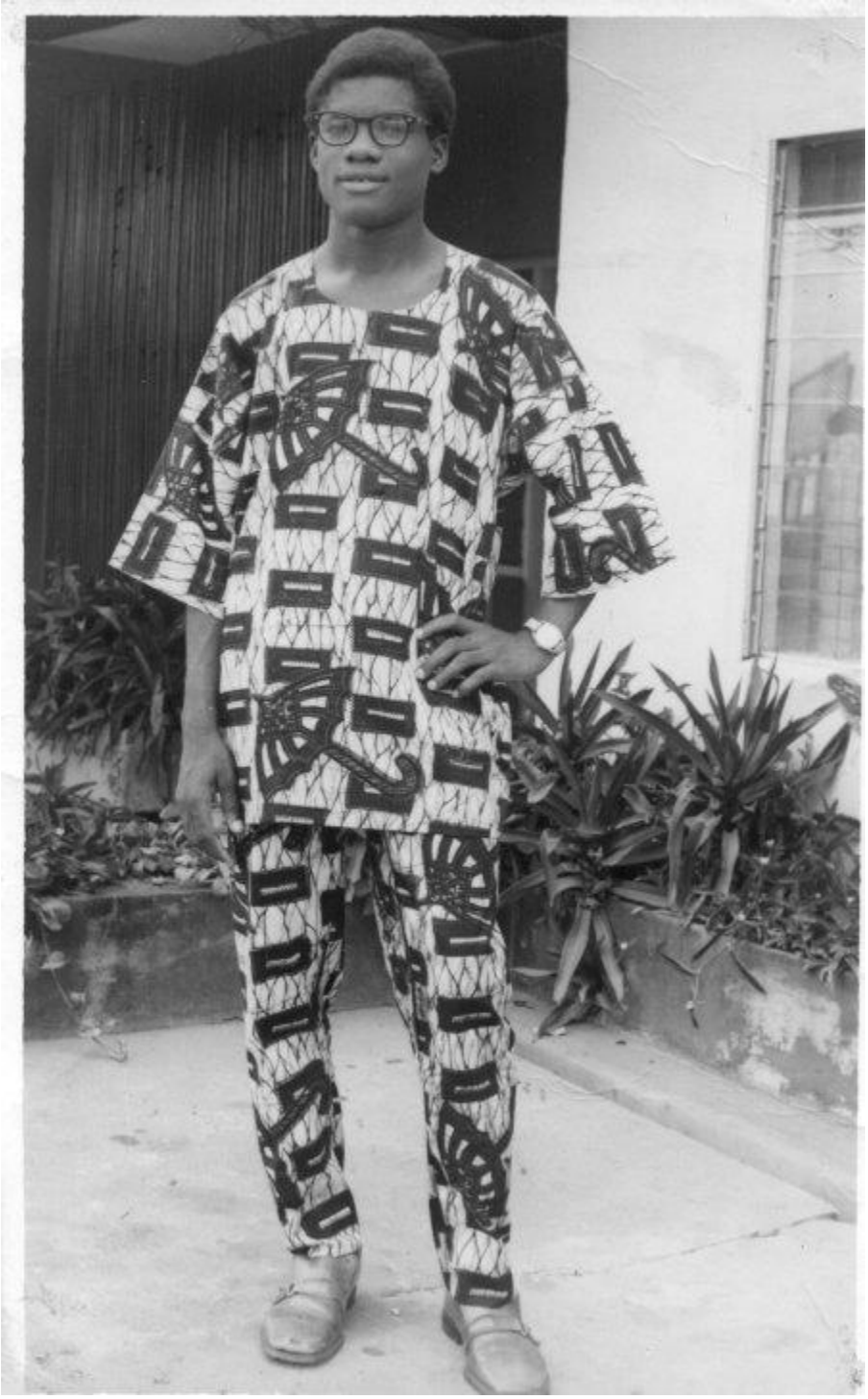
$$= \nabla \cdot \overleftrightarrow{K} \cdot \left[\frac{R_{so}\lambda_o}{B_o}\nabla\Phi_o + \frac{R_{sw}\lambda_w}{B_w}\nabla\Phi_w + \frac{\lambda_g}{B_g}\nabla\Phi_g\right] - \frac{q_g}{\rho_{gsc}}$$

HYPERBOLIC EQUATIONS

The nine equations derived by Emeagwali are used, instead of Darcy's formula. These nine equations are solved simultaneously with the conservation of mass and several other equations.

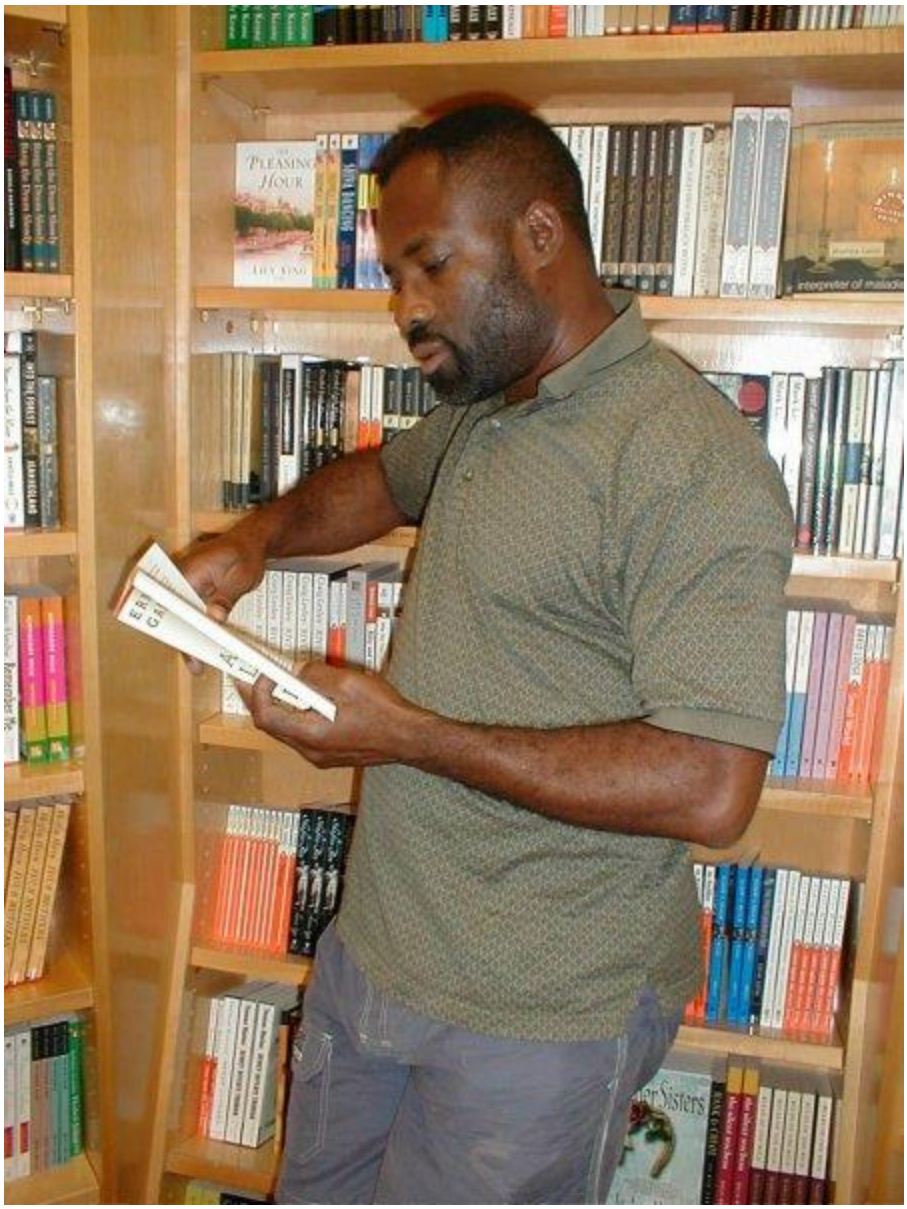
Emeagwali (4th from left), a few weeks before leaving Nigeria to study in the United States.
(Peter Ozoh, Umeano, Frank Tagbo, Emeagwali, Lawrence, Christ the King College, Onitsha, Nigeria. January 1974)

Emeagwali
(5 Oguta Road, Onitsha, Nigeria. Mid-1973)

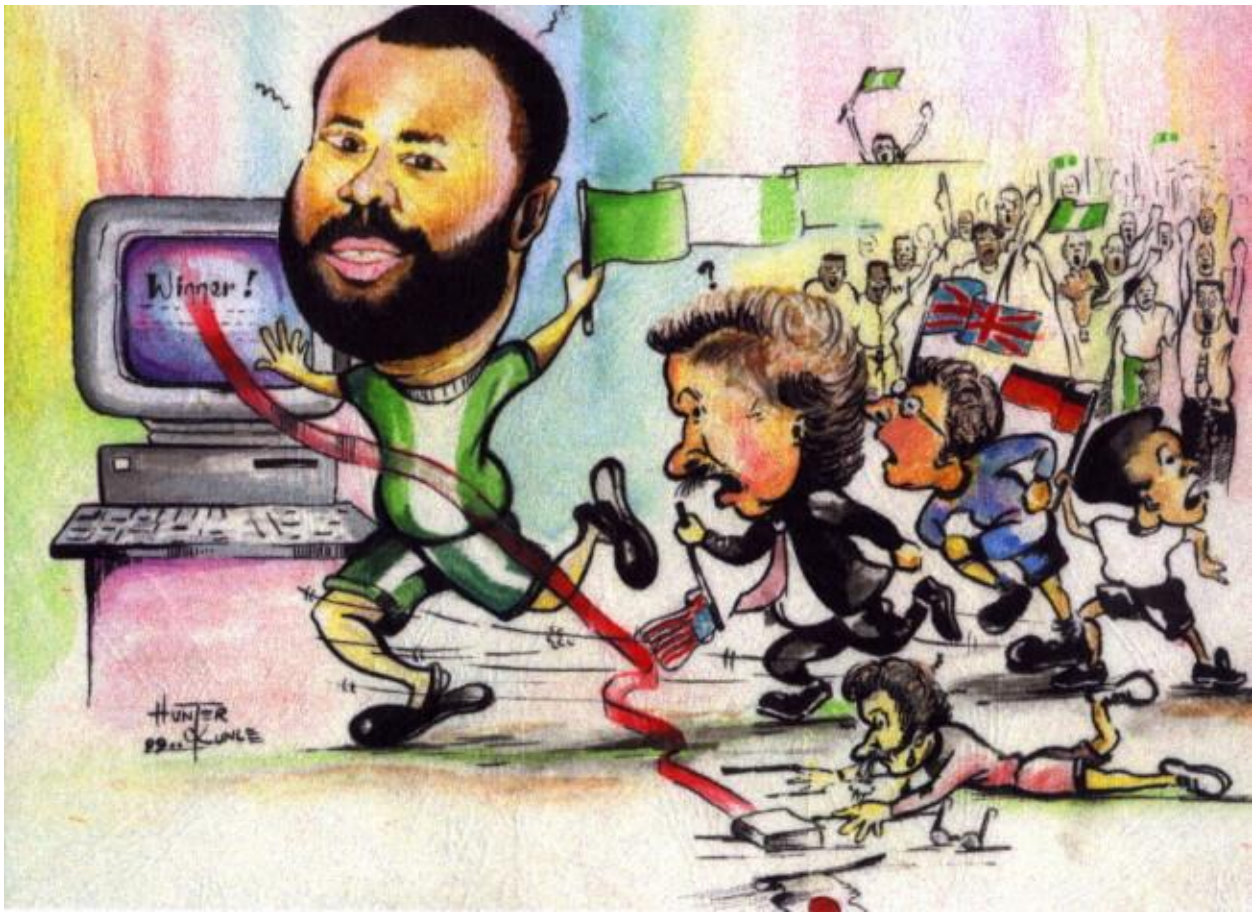


In Nov-Dec 1972, I came to Enugu to take my Scholastic Aptitute Test

(SAT), Achievement Test (in mathematics, physics and chemistry) and Test Of English as a Foreign Language (TOEFL) examinations. A few days later, I returned to Onitsha to take my University of London General Certificate of Education examinations as an external candidate. The clothes that I had is called *dansiki* (pronounced "dahn-SEE-kee"). It was sowned by an itinerant "Obioma" tailor from my mother's *lappa*. (Emeagwali, photo taken at studio at 1 & 3 Ajalli Street, Uwani, Enugu, Nigeria. 1972. I lived at 41 Bishop Ayogu Road, Uwani)



Emeagwali at a bookstore.
(Baltimore, Maryland. October 21, 2000)



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Number:	76

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